[0521] Note the similarities for the planar hexagonal sensor of **FIG. 15**(*b*) where Y is the coordinate measured by the two sensor subsystems using transducers **1502**, **1511**, **1508** and **1505**, U is the coordinate measured by the two sensor subsystems using transducers **1504**, **1501**, **1510** and **1507**, V is the coordinate measured by the two sensor subsystems using transducers **1512**, **1509**, **1506** and **1503**, and the center of the sensor corresponds to X=Y=U=V=0.

$$U=-\sqrt{3}X/2+Y/2 \quad \text{or} \quad X=+Y/\sqrt{3}-2U/\sqrt{3}$$

$$V=\sqrt{3}X/2+Y/2 \quad \text{or} \quad X=-Y/\sqrt{3}+2V/\sqrt{3}$$

$$X=(-U+V)/\sqrt{3}$$

$$Y=U+V$$

[0522] There is a quantitative analogy between X, Y, U, and V with $\delta\theta$, $\delta\phi$, δu, and δv in the small quasi-flat region at the top of the sensor of **FIG. 21**(*d*). In this sense, The sensor of **FIG. 21**(*d*) is a non-planar generalization of the sensor of **FIG. 15**(*b*). Similarly, there are non-planar generalizations of other planar sensors geometries.

[0523] Reflector spacing and angles can be calculated using previously discussed principles. Let us refer again to this first sensor subsystem in **FIG. 21**(*d*). For the transmit array, the reflector spacing vector is $S=2\pi n(k^t(s)-k_p(s))/|k_t(s)-k_p(s)|^2$ where $k_t(s)$ and $k_p(s)$ can be calculated from the known array trajectory ($\theta(s)$, $\phi(s)$) given above by the following expressions.

$$k_t(s)=(2\pi/\lambda)\cdot(-\sin(\pi s/2), \quad \sin(\Theta)\cdot\cos(\pi s/2), \quad \cos(\Theta)\cdot\cos(\pi s/2))$$

$$k_p(s)=(2\pi/\lambda')\cdot(-\cos(\phi(s))\sin(\theta(s)), \quad -\sin(\phi(s))\sin(\theta(s)), \quad \cos(\theta(s)))$$

[0524] Here λ represents the wavelength of the acoustic mode traveling along the transmit array and λ' represents the wavelength traversing the touch region.

[0525] Note that the maximum acoustic path length for the sensor subsystems is $(2\pi-2\Theta)R$. For $\Theta=20°$, this becomes 5.585*R. For Rayleigh waves at 5.53 MHz on aluminum or borosilicate glass substrates, this means the dome sensor can have a radius in excess of 10 inches. Even larger sizes are possible if a lower operating frequency is used, or other means are provided to reduce acoustic attenuation or tolerate weaker signal amplitudes.

[0526] Therefore, an application for the hemispheric dome sensor according to this embodiment is, for example, in an interactive museum environment. For example, a 20 inch (or a ½ meter) diameter borosilicate-glass dome sensor with a reverse-projection screen laminated on the back side may be provided. Star patterns of the night sky, or a section of the Earth's globe may be projected onto the sensor. This system could support a table-top hands-on planetarium or an interactive globe exhibit. The touch surface, arrays, and transducers may be placed on the concave side of the sensor; for example, an interactive touch sensitive aquarium portal may be provided, perhaps in combination with an ultrasonic fish finding/identification system, in which the user points to sea creatures that may swim by. Many other applications can be imagined.

Example 15

[0527] FIGS. **22**(*a*) and **22**(*b*) provide an example which illustrates the inherent geometric flexibility of the present invention. It shows a basin which may be thought of as a flattened and otherwise distorted hemisphere with a hole in it for a drain. Such a sensor geometry may be of interest as a basin perhaps containing a liquid. Furthermore the touch sensitive surface is on the inside rather than the outside. The reflective arrays **2201**, **2202**, **2203**, **2204**, **2205**, **2206** are disposed with topological similarity to the hexagonal sensor of **FIG. 15**(*b*); there are six superposed arrays, each with one transmit **2207**, **2208**, **2209**, **2210**, **2211**, **2212** and one receive transducer **2213**, **2214**, **2215**, **2216**, **2217**, **2218**. As with the hexagonal sensor of **FIG. 15**(*b*), a third set of reflectors may be superposed on each array to support sensor subsystems involving opposite pairs of arrays.

[0528] The sides of the basin **2200** are vertical at the locations of the transducers and arrays. Thus the intersection of a horizontal plane at the level of the transducers and arrays with the basin forms a geodesic loop. The acoustic paths along the arrays follow sections of this closed loop geodesic.

[0529] For the transmit and receive arrays of each sensor subsystem, we define the path parameter "s" to be arc length of the path along the array from the transducer an ay divided by the total arc length of the array.

[0530] Conceptually, the geodesic paths across the touch region for value s can be determined as follows. A string is anchored on the transmit array corresponding to the value of the path length parameter s. The string is looped over the convex surface of the basin **2200** so that it intersects the receive array at the location corresponding to the path length parameter s. The string is pulled taught, and the length of the string between the arrays and the directions of the string at both arrays is observed; this determines the path length of the geodesic across the touch sensitive zone as well as the directions of the wave vectors of the geodesic where scattering takes place on the transmit and receive arrays. In practice, this conceptual scheme serves as the mathematical basis of a computer simulation algorithm that solves the acoustic path geometry. In this manner, all relevant geometric information of **FIG. 20** may be determined.

[0531] If the basin **2200** is very deep, e.g. not a flattened hemisphere but rather a stretched hemisphere, then the geodesics between the arrays might not pass through the desired touch region. In the string analogy, pulling the string tight may cause the string to slip off the desired touch zone. In this case, the design engineer can either flatten the basin geometry or introduce intermediate scatterings in the acoustic path with reflective boundaries.

[0532] For any choice of acoustic modes and substrate options, the principles of **FIG. 20** and the spacing-vector formula allow calculation of reflector angles and spacings. As is typical of present commercial acoustic touchscreen design methods, modulation of array reflectivity (e.g., via reflector density, reflector height, or line width) can be determined iteratively by building prototypes, observing signal uniformity (or lack thereof), and improving the modulation of array reflectivity. Means are thus available to design reflector arrays for the sensor in FIGS. **22**(*a*) and **22**(*b*).